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RP-166: Solving Some Special Standard Cubic Congruence of Composite Modulus modulo a Multiple of an Odd Prime

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ABSTRACT

Here in this paper, ten special type of standard cubic congruence of composite modulus are studied for their solutions. It is found that each of the cubic congruence under consideration has a single solution. The solution can be obtained orally as the solution is given in the problems. No extra effort is necessary to find the solution.

KEYWORDS: *Cubic Congruence, Composite Modulus, Unique Solution*

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INTRODUCTION

Some standard cubic congruence of special type are considered for study and are formulated their solutions. All the considered cubic congruence have unique solutions.

Those solutions are present in the congruence itself. Here is the list of those cubic Congruence in the problem statement.

PROBLEM-STATEMENT

"To find formula for solutions of the congruence:

$x^3 \equiv p \ (mod\ 2p)$,

$x^3 \equiv p \ (mod\ 3p)$,

$x^3 \equiv 2p \ (mod\ 3p)$,

$x^3 \equiv p \ (mod\ 4p)$,

$x^3 \equiv 3p \ (mod\ 4p)$,

$x^3 \equiv p \ (mod\ 6p)$,

$x^3 \equiv 2p \ (mod\ 6p)$,

$x^3 \equiv 3p \ (mod\ 6p)$,

$x^3 \equiv 4p \ (mod\ 6p)$,

$x^3 \equiv 5p \ (mod\ 6p), p\ being\ an\ odd\ prime."$

LITERATURE REVIEW

The standard cubic congruence found no place in the literature of mathematics as it is not studied; it is not a part of syllabus in the university course. Only linear congruence of degree one and standard quadratic congruence of prime and composite modulus are remained in the part of study [1], [2], [3].Also some of the author's papers are seen [4], [5], [6].

ANALYSIS & RESULTS

Consider the congruence: $x^3 \equiv p \ (mod\ 2p)$. Here p is an odd prime.

It is easily seen that: $p^3 - p = p(p^2 - 1) = p.2t \equiv 0 \ (mod\ 8p)$ as $p\ is\ odd$.

Hence $x \equiv p \ (mod\ 2p)$ is a solution of the congruence.

Consider the congruence: $x^3 \equiv p \ (mod\ 3p)$. Here p is an odd prime.

It is easily seen that: $p^3 - p = p(p-1)(P+1) = p.3t \equiv 0 \ (mod\ 3p)$ as $p\ is\ odd$.

Hence $x \equiv p \ (mod\ 3p)$ is a solution of the congruence.

Consider the congruence: $x^3 \equiv 2p \ (mod\ 3p)$. Here p is an odd prime.

It is easily seen that: $(2p)^3 - 2p = 2p(2p-1)(2P+1) = p.3t \equiv 0 \ (mod\ 3p)$ as $p\ is\ odd$.

Hence $x \equiv 2p \ (mod\ 3p)$ is a solution of the congruence.

Consider the congruence: $x^3 \equiv p \ (mod\ 4p)$. Here p is an odd prime.

It is easily seen that: $p^3 - p = p(p^2 - 1) = p(p-1)(p+1) = p.4t \equiv 0 \ (mod\ 4p)$ as $p\ is\ odd$.

Hence $x \equiv p\ (mod\ 4p)$ is a solution of the congruence.

Consider the congruence: $x^3 \equiv 3p\ (mod\ 4p)$. Here p is an odd prime.

It is easily seen that: $(3p)^3 - 3p = 3p(9p^2 - 1) = 3p[4p^2 + 4p^2 + (p^2 - 1)]$

$= 3p.4t \equiv 0\ (mod\ 4p)$.

Hence $x \equiv 3p\ (mod\ 4p)$ is a solution of the congruence.

Consider the congruence: $x^3 \equiv p\ (mod\ 8p)$. Here p is an odd prime.

It is easily seen that: $p^3 - p = p(p^2 - 1) = p.8t \equiv 0\ (mod\ 8p)$ as $p^2 \equiv 1\ (mod\ 8)$.

Hence $x \equiv p\ (mod\ 8p)$ is a solution of the congruence

Consider the congruence: $x^3 \equiv 3p\ (mod\ 8p)$. Here p is an odd prime.

It is easily seen that: $(3p)^3 - 3p = 3p(9p^2 - 1) = 3p[8p^2 + (p^2 - 1)]$

$= 3p.8t \equiv 0\ (mod\ 8p)$.

Hence $x \equiv p\ (mod\ 8p)$ is a solution of the congruence.

Consider the congruence:$x^3 \equiv 5p\ (mod\ 8p)$. Here p is an odd prime.

It is easily seen that: $(5p)^3 - 5p = 5p(25p^2 - 1) = 5p[24p^2 + (p^2 - 1)]$

$= 5p.8t \equiv 0\ (mod\ 8p)$.

Hence $x \equiv 5p\ (mod\ 8p)$ is a solution of the congruence.

Consider the congruence: $x^3 \equiv 7p\ (mod\ 8p)$. Here p is an odd prime.

It is easily seen that: $(7p)^3 - 7p = 7p(49p^2 - 1) = 7p[48p^2 + (p^2 - 1)]$

$= 7p.8t \equiv 0\ (mod\ 8p)$.

Hence $x \equiv 7p\ (mod\ 8p)$ is a solution of the congruence.

ILLUSTRATIONS

Example-1: Consider the congruence $x^3 \equiv 7\ (mod\ 14)$.

It can be written as $x^3 \equiv 7\ (mod\ 2.7)$.

It is of the type $x^3 \equiv p\ (mod\ 2p)\ with\ p = 7$.

It has single solution $x \equiv p\ (mod\ 2p)$

$\equiv 7\ (mod\ 2.7)$

$\equiv 7\ (mod\ 14)$.

Consider the congruence $x^3 \equiv 7\ (mod\ 21)$.

It can be written as $x^3 \equiv 7\ (mod\ 3.7)$.

It is of the type $x^3 \equiv p\ (mod\ 3p)\ with\ p = 7$.

It has single solution $x \equiv p\ (mod\ 3p)$

$\equiv 7\ (mod\ 3.7)$

$\equiv 7\ (mod\ 21)$.

Consider the congruence $x^3 \equiv 14\ (mod\ 21)$.

It can be written as $x^3 \equiv 2.7\ (mod\ 3.7)$.

It is of the type $x^3 \equiv 2p\ (mod\ 3p)\ with\ p = 7$.

It has single solution $x \equiv 2p\ (mod\ 3p)$

$\equiv 2.7\ (mod\ 3.7)$

$\equiv 14\ (mod\ 21)$.

Example-1: Consider the congruence $x^3 \equiv 7\ (mod\ 56)$.

It can be written as $x^3 \equiv 7\ (mod\ 8.7)$.

It is of the type $x^3 \equiv p\ (mod\ 8p)\ with\ p = 7$.

It has single solution $x \equiv p\ (mod\ 8p)$

$\equiv 7\ (mod\ 8.7)$

$\equiv 7\ (mod\ 56)$.

Example-2: Consider the congruence $x^3 \equiv 21\ (mod\ 56)$.

It can be written as $x^3 \equiv 3.7\ (mod\ 8.7)$.

It is of the type $x^3 \equiv 3p\ (mod\ 8p)\ with\ p = 7$.

It has single solution $x \equiv 3p\ (mod\ 8p)$

$\equiv 3.7\ (mod\ 8.7)$

$\equiv 21\ (mod\ 56)$.

Example-3: Consider the congruence $x^3 \equiv 35\ (mod\ 56)$.

It can be written as $x^3 \equiv 5.7\ (mod\ 8.7)$.

It is of the type $x^3 \equiv 5p\ (mod\ 8p)\ with\ p = 7$.

It has single solution $x \equiv 5p\ (mod\ 8p)$

$\equiv 5.7\ (mod\ 8.7)$

$\equiv 35\ (mod\ 56)$.

Example-4: Consider the congruence $x^3 \equiv 49\ (mod\ 56)$.

It can be written as $x^3 \equiv 7.7\ (mod\ 8.7)$.

It is of the type $x^3 \equiv 7p\ (mod\ 8p)\ with\ p = 7$.

It has single solution $x \equiv 7p\ (mod\ 8p)$

$\equiv 7.7\ (mod\ 8.7)$

$\equiv 49\ (mod\ 56)$.

CONCLUSION

It can be concluded from this discussion that the standard cubic congruence considered, each has a single solutions.

It is found that the congruence $x^3 \equiv p\ (mod\ 2p)$, p an odd prime has a unique solution

$$x \equiv p\ (mod\ 2p).$$

The congruence $x^3 \equiv p\ (mod\ 3p)$, p an odd prime has a unique solution

$$x \equiv p\ (mod\ 3p).$$

The congruence $x^3 \equiv 2p\ (mod\ 3p)$, p an odd prime has a unique solution

$$x \equiv 2p\ (mod\ 3p).$$

The congruence $x^3 \equiv p\ (mod\ 4p)$, p an odd prime has a unique solution

$$x \equiv p\ (mod\ 4p).$$

The congruence $x^3 \equiv 3p\ (mod\ 4p)$, p an odd prime has a unique solution

$$x \equiv 3p\ (mod\ 4p).$$

The congruence $x^3 \equiv p\ (mod\ 8p)$, p an odd prime has a unique solution

$$x \equiv p \ (mod\ 8p).$$

The congruence $x^3 \equiv 3p \ (mod\ 8p)$, p an odd prime has a unique solution

$$x \equiv 3p \ (mod\ 8p).$$

The congruence $x^3 \equiv 2p \ (mod\ 3p)$, p an odd prime has a unique solution

$$x \equiv 2p \ (mod\ 3p).$$

The congruence $x^3 \equiv 5p \ (mod\ 8p)$, p an odd prime has a unique solution

$$x \equiv 5p \ (mod\ 8p).$$

The congruence $x^3 \equiv 7p \ (mod\ 8p)$, p an odd prime has a unique solution

$$x \equiv 7p \ (mod\ 8p).$$

MERIT OF THE PAPER

The use of Chinese remainder theorem is needless. Solutions can be obtained orally. This is the merit of the paper.

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